investigation on a kaonic nuclear production reaction. The possible reactions are $d(p,K^0p)ppK^-$, $^3He(K^-,\pi^-)ppK^-$, $^3He(\pi^+,K^+)ppK^-$, $d(p,K^+)K^-ppn$, $d(p,K^0)K^-ppp$, $^3He(p,K^+)K^-ppn$, $^3He(p,K^0)K^-ppp$.

Among them, we choose the first reaction for our investigation, since our interest lies in the lightest kaonic nuclei namely ppK^- and deuteron target is simpler than helium target.

II. Formulation of Differential Cross Section

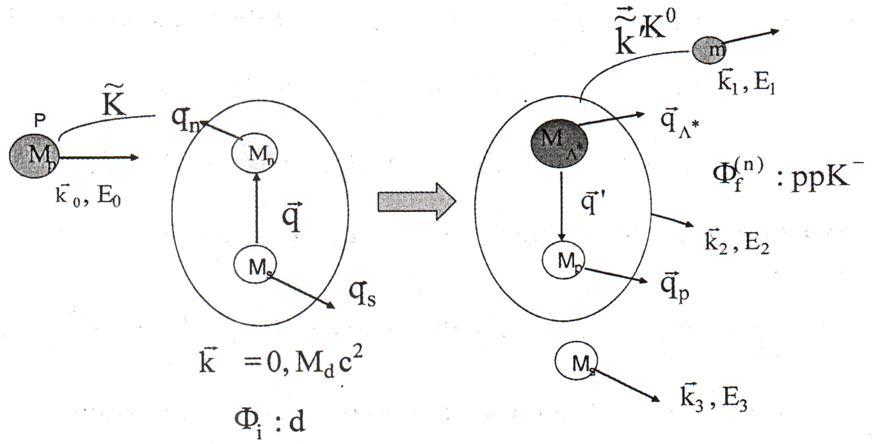


Figure (1) Schematic diagram of $d(p, K^0p)ppK^-$ reaction

The behavior of a reaction is usually expressed in terms of a differential cross section. The differential cross section is defined as the ratio of transition rate to incident flux. We introduce the transition rate from Fermi's Golden rule

$$W_{fi} = \frac{2\pi}{\hbar}|T_{fi}|^2\, \delta(E_i - E_f^n)\left(\frac{L}{2\pi}\right)^3 dk_1 \left(\frac{L}{2\pi}\right)^3 dk_2 \left(\frac{L}{2\pi}\right)^3 dk_3 \tag{1}$$

where $|T_{fi}|^2$= transition probability and $\delta(E_f^n - E_i)$ is energy conservation term.

Differential cross section of reaction is

$$d^9\sigma = \frac{L^3}{v_0}\frac{2\pi}{\hbar}\delta(E_f^n - E_i)\left(\frac{L}{2\pi}\right)^3 dk_1\left(\frac{L}{2\pi}\right)^3 dk_2\left(\frac{L}{2\pi}\right)^3 dk_3 \times |T_{fi}|^2. \tag{2}$$

The transition matrix from initial state to final state of reaction is written as

$$T_{fi} = \frac{\hbar c}{\sqrt{2E_1}}\left[k_1, k_2, \Phi_f^n, k_3 \middle| T \middle| k_0, k_d = 0, \Phi_i\right] \tag{3}$$

We used completeness relation to distinguish centre of mass and relative momentum and we obtain

$$T_{fi} = \frac{\hbar c}{\sqrt{2E_1}}\left(\frac{L}{2\pi}\right)^{12}\int dq_p\, dq_{\Lambda^*}\, dq_n\, dq_s \left[\Phi_f^n, k_2 \middle| q_p, q_{\Lambda^*}\right] \left[q_p, q_{\Lambda^*}, k_1, k_3 \middle| T \middle| k_0, q_n, q_s\right]\left[q_n, q_s \middle| k_d = 0, \Phi_i\right] \tag{4}$$

By explaining equation(4), the transition matrix is got as

$$T_{fi} = \frac{\hbar c}{\sqrt{2E_1}}\delta(k_1 + k_2 + k_3 - k_0)\int dq' \left\langle \Phi_f^n \middle| q'\right]\left[\tilde{k}', q' \middle| t \middle| \tilde{k}\right]\left[q \middle| \Phi_i\right\rangle \tag{5}$$

The transition matrix in coordinate representation is written as

$$T_{fi} = \frac{\hbar c}{\sqrt{2E_1}}\left(\frac{2\pi}{L}\right)^{\frac{15}{2}} (\mathbf{k}_0 - \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3) \tag{6}$$

$$\int dq' \int dr \int dr' \langle \Phi_f^n | q'] [\tilde{k}', q' | r \rangle \langle r | t | r' \rangle \langle r' | \tilde{k}] \; [-k | \Phi_i \rangle$$

We change the transition matrix from coordinate space to momentum space.

$$T_{fi} = \frac{\hbar c}{\sqrt{2E_1}} \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 - \mathbf{k}_0) \left(\frac{2\pi}{L}\right)^{\frac{15}{2}} \int dq' \langle \Phi_f^n | q' \rangle \langle \tilde{k}', q' | t | \tilde{k} \rangle \langle -\mathbf{k}_3 | \Phi_i \rangle. \tag{7}$$

By substituting equation(7) into equation (2), the differential cross section is obtained as

$$d^9\sigma = 2\pi L^3 \frac{E_0}{2k_0 E_1} \delta(E_i - E_f^n) \left(\frac{L}{2\pi}\right)^9 \iiint dk_1 dk_2 dk_3 \left(\frac{2\pi}{L}\right)^{15} \tag{8}$$

$$\left| \int dq' \langle \Phi_f^n | q' \rangle \langle \tilde{k}', q' | t | \tilde{k} \rangle \right|^2 \left| \langle -\mathbf{k}_3 | \Phi_i \rangle \right|^2 \{\delta(\mathbf{k}_0 - \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3)\}^2 .$$

By carrying out $\int dk_2$ the differential cross section of reaction is

$$d^6\sigma = (2\pi)^4 \frac{E_0}{2k_0 E_1} \delta(E_i - E_f^n) \left(\frac{L}{2\pi}\right)^9 \iint dk_1 dk_3 \left| \int dq' \langle \Phi_f^n | q' \rangle \langle \tilde{k}', q' | t | \tilde{k} \rangle \right|^2 \left| \langle -\mathbf{k}_3 | \Phi_i \rangle \right|^2 \tag{9}$$

$\delta(E_i - E_f^n)$ is energy conservation term. It is solved in the following..

The total energies of the initial state and all final states are

$$E_i = E_0 + M_d c^2 . \tag{10}$$

$$E_f^n = E_1 + E_3 + \sqrt{M_{ppk^-}^2 c^4 + (\hbar c)^2 (\mathbf{k}_0 - \mathbf{k}_1 - \mathbf{k}_3)^2} . \tag{11}$$

$$E_f^n = E_1 + E_3 + \sqrt{M^2_{ppk^-} c^4 + \hbar^2 c^2 \mathbf{k}^2} \tag{12}$$

$$E_f^{(n)} \approx E_1 + E_3 + M^{(n)}_{ppk^-} c^2 + \overline{M}_{ppk^-} c^2 \left\{ \sqrt{1 + \frac{\hbar^2 k_2^2}{\overline{M}^2_{ppk^-} c^2}} - 1 \right\} \tag{13}$$

$$E_f^{(n)} = E_1 + E_3 + M_{\Lambda^*} c^2 + M_p c^2 + \langle \Phi_f^n | H_{\Lambda^* p} | \Phi_f^n \rangle + E_{recoil} \tag{14}$$

where $E_{recoil} = \overline{M}_{ppk^-} c^2 \left\{ \sqrt{1 + \frac{\hbar^2 k_2^2}{\overline{M}^2_{ppk^-} c^2}} - 1 \right\}$, $\overline{M}_{ppk^-} = 2M_p + m.$

The energy difference between initial state and final state is

$$E_i - E_f^n = E - \langle \Phi_f^n | H_{\Lambda^* p} | \Phi_f^n \rangle \tag{15}$$

where $$E \approx (E_0 - E_1 - E_3) + (M_d - M_{\Lambda^*} - M_p) c^2 - E_{recoil} \tag{16}$$

and E gives total energy of $\Lambda^* p$. Therefore the energy conservation term can be rewritten as

$$\delta(E_i - E_f^n) = \langle \Phi_f^n | \delta(E - H_{\Lambda^* p}) | \Phi_f^n \rangle . \tag{17}$$

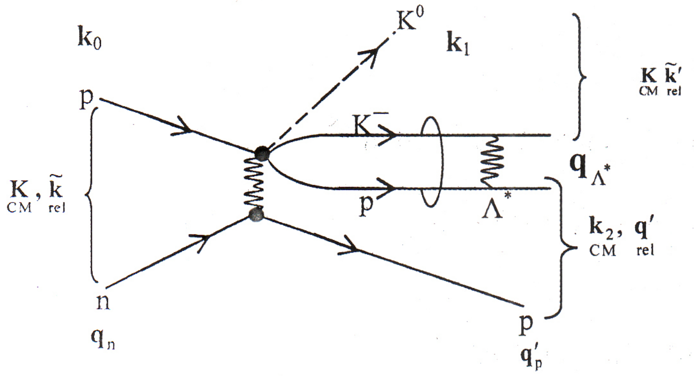
By using delta function properties the energy conservation term becomes

$$\delta(E_i - E_f^n) = -\frac{1}{\pi}\mathrm{Im}\left\langle \Phi_f^n \left| \frac{1}{E-H_{\Lambda^* p}+i\varepsilon}\right| \Phi_f^n\right\rangle. \tag{18}$$

$$\sum_n \delta(E_i - E_f^n)\left|\int dq' \langle\Phi_f^n|q'\rangle\langle\tilde{k}',q'|t|\tilde{k}\rangle\right|^2 |\langle -k_3|\Phi_i\rangle|^2$$
$$= -\frac{1}{\pi}\mathrm{Im}\left[\iint dq''dq'\langle\tilde{k}|t^*|\tilde{k}',q''\rangle\times\left\langle q''\left|\frac{1}{E-H_{\Lambda^* p}+i\varepsilon}\right|q'\right\rangle\langle\tilde{k}',q'|t|\tilde{k}\rangle\right]|\langle -k_3|\Phi_i\rangle|^2 \tag{19}$$

$$d^6\sigma = (2\pi)^4\frac{E_0}{2k_0E_1}\iint dk_1\,dk_3|\langle -k_3|\Phi_i\rangle|^2\,(-\frac{1}{\pi})\times\mathrm{Im}\left[\iint dr\,dr'\langle\tilde{k}|t^*|\tilde{k}',r\rangle\langle r|G|r'\rangle\langle\tilde{k}',r'|t|\tilde{k}\rangle\right] \tag{20}$$

The final term of the above equation in square bracket is transition matrix of elementary process. To solved the transition matrix of elementary process we consider the elementary process of $p(d,K^0p)ppK^-$ reaction as $p+n\rightarrow(pK^-)+K^0+p\rightarrow\Lambda^*+K^0+p$



The $p \longrightarrow p + K^0 + K^-$ process, where K^0K^- pair is assumed to be created at zero range from a proton, is of highly off-energy shell $(\Delta E \approx 2m_K)$. This process is realized with a large momentum transfer to the neutron, which is done efficiently by the p-n short-range interaction. This interaction is expressed by $e^{\frac{(-m_B r)}{r}}$ with m_B being the intermediate boson mass.

The interaction matrix element for $\Lambda^* p$ formation is

$$\langle\tilde{k}',r'|t|\tilde{k}\rangle = \int dq'\langle r'|q'\rangle\ \langle q',\tilde{k}',\phi_{\Lambda^*}|t|\tilde{k}\rangle. \tag{21}$$

It can be separated into five terms as

$$\langle q',\tilde{k}',\phi_{\Lambda^*}|t|\tilde{k}\rangle = \iint\iint dr_{K^0\Lambda^*p}dr_{p\Lambda^*}dr_{pK^-}dr_{pn}\langle\tilde{k}'|r_{K^0\Lambda^*p}\rangle\langle q'|r_{p\Lambda^*}\rangle$$
$$\times\langle\varphi_{\Lambda^*}|r_{pK^-}\rangle\langle r_{pn}|\tilde{k}\rangle\times\langle r_{K^0\Lambda^*p},r_{p\Lambda^*},r_{pK^-}|t|r_{pn}\rangle. \tag{22}$$

The 5th term of equation (22), the effective interaction for the elementary process is expressed as

$$\langle r_{K^0\Lambda^*p},r_{p\Lambda^*},r_{pK^-}|t|r_{pn}\rangle = t_0\iiint dr E(r)\ \delta(r_{K^0\Lambda^*p}-\eta r)\delta(r_{p\Lambda^*}-r)\delta(r_{pK^-})\delta(r_{pn}-r) \tag{23}$$

where, $F(r)=\frac{\beta}{r}e^{-\frac{r}{\beta}}$, $\beta=\frac{\hbar c}{m_\beta c^2}$, $\eta=\frac{M_p}{M_{ppk^-}}$.

By substituting equation (23) into (22), we get the following equation.

$$\langle\tilde{\mathbf{k}}',\mathbf{q}',\varphi_{\Lambda^*}|t|\tilde{\mathbf{k}}\rangle=t_0\,\varphi_{\Lambda^*}(0)\int d\mathbf{r}\frac{\beta}{r}e^{-\frac{r}{\beta}}\langle\tilde{\mathbf{k}}'|\eta\mathbf{r}\rangle\langle\mathbf{q}'|\mathbf{r}\rangle\langle\mathbf{r}|\tilde{\mathbf{k}}\rangle \tag{24}$$

Equation (21) becomes

$$\langle\tilde{\mathbf{k}}',\mathbf{r}'|t|\tilde{\mathbf{k}}\rangle=t_0\,\varphi_{\Lambda^*}(0)\frac{\beta}{r'}e^{-\frac{r'}{\beta}}\langle\tilde{\mathbf{k}}'|\eta\mathbf{r}'\rangle\langle\mathbf{r}'|\tilde{\mathbf{k}}\rangle. \tag{25}$$

$$\langle\tilde{\mathbf{k}}',\mathbf{r}'|t|\tilde{\mathbf{k}}\rangle=\bar{v}_0\frac{1}{r}e^{-\frac{r'}{\beta}}e^{i\mathbf{Q}\mathbf{r}'}\equiv\bar{v}_0 f(\mathbf{r}') \tag{26}$$

where, $\bar{v}_0=\frac{1}{(2\pi)^3}t_0\left(\frac{2}{\pi\alpha^2}\right)^{\frac{3}{4}}\beta$, $f(\mathbf{r}')=\frac{1}{r'}e^{-\frac{r'}{\beta}}e^{i\mathbf{Q}\mathbf{r}'}$

$\mathbf{Q}=-\eta\tilde{\mathbf{k}}'+\tilde{\mathbf{k}}=\eta_0\mathbf{k}_0-\eta_1\mathbf{k}_1+\eta_3\mathbf{k}_3$, $\eta_0=1-\frac{M_p}{M_n+M_p}+\frac{m}{M_{ppK}}\frac{M_p}{M_{ppK}}=1-\eta_3$, $\eta_1=\frac{M_p}{M_{ppK}}$,

$\eta_3=\frac{M_p}{M_n+M_p}-\frac{m}{M_{ppK}}\frac{M_p}{M_{ppK}}$

From equation (20) and (26), the reaction differential cross section is

$$d^6\sigma=(2\pi)^4\frac{E_0}{2k_0E_1}\iint dk_1\,dk_3\left|\langle-\mathbf{k}_3|\Phi_i\rangle\right|^2|\bar{v}_0|^2\times\left(-\frac{1}{\pi}\right)\,\mathrm{Im}\left[\iint d\mathbf{r}\,d\mathbf{r}'f^*(\mathbf{r})\langle\mathbf{r}|\frac{1}{E-H_{\Lambda^*p}+i\varepsilon}|\mathbf{r}'\rangle f(\mathbf{r}')\right] \tag{27}$$

The production differential cross section of $\Lambda^*p(=ppK^-)$ is

$$\frac{d^6\sigma}{dE_1d^2\Omega_1dE_3d^2\Omega_3}=\left(\frac{2\pi}{\hbar c}\right)^4|\bar{v}_0|^2\frac{k_1k_3E_0E_3}{2k_0}\left|\langle-\mathbf{k}_3|\Phi_i\rangle\right|^2\left(\frac{-1}{\pi}\right)\,\mathrm{Im}\left[\iint d\mathbf{r}\,d\mathbf{r}'f^*(\mathbf{r})\langle\mathbf{r}|\frac{1}{E-H_{\Lambda^*p}+i\varepsilon}|\mathbf{r}'\rangle f(\mathbf{r}')\right] \tag{28}$$

In the above equation $\langle\mathbf{r}|\frac{1}{E-H_{\Lambda^*p}+i\varepsilon}|\mathbf{r}'\rangle$ is the Green's function $G(\mathbf{r},\mathbf{r}')$ It can be separated into radial part and angular part as

$$G(\mathbf{r},\mathbf{r}')=\sum_{l=0}^{\infty}\sum_{M}Y_{lM}(\hat{r})\frac{G(r,r')}{rr'}Y^*_{lM}(\hat{r}') . \tag{29}$$

The radial part satisfies the following equation,

$$\left\{k^2+\frac{d^2}{dr^2}-\frac{\ell(\ell+1)}{r^2}-\tilde{V}(r)\right\}G(r,r')=\frac{2\mu}{\hbar^2}\delta(r-r') \tag{30}$$

where $k=\sqrt{\frac{2\mu E}{\hbar^2}}$, $\tilde{V}(r)=\frac{2\mu}{\hbar^2}U_{opt}(r)$.